

The Use of LP Simplex Method in the Determination of the Minimized Cost of a Newly Developed Core Binder

Paul Aondona IHOM¹, Jacob JATAU² and Hamisu MUHAMMAD²

¹*National Metallurgical Development Centre, Jos Plateau State-Nigeria*

²*Abubakar Tafawa Belewa University, Bauchi, Nigeria*

paulihom@yahoo.co.uk, jacjatau@yahoo.co.uk, mmhamisu@yahoo.com

Abstract

A new binder system has been developed which is based on manihot esculenta and cement. The new binder system is expected to be an alternative to Urea formaldehyde furfuryl alcohol binder system commonly used in the foundry for core making; it is a cold setting binder system. The cost factor is one of the reasons for the development of this new binder system, therefore LP simplex method model was used in the determination of the minimized cost using this newly developed binder system for core making. The result showed that the cost was \$1.57/kg of core mixture as against \$5 for using urea formaldehyde furfuryl alcohol binder system.

The work also revealed that strict adherence to the compositional constraints is very important since any change in either of the constraints would lead to a change in the total cost of producing the core mixture and consequently too, a change in the properties of the cores produced using the core mixture.

Keywords

Application; LP Simplex Method Model; Developed; Core Making.

Introduction

Casting is one production process that is efficient and has minimal losses in terms of materials utilization [1].

Foundry, which is a place where casting takes place, is yet to be a sector with a serious investment in Nigeria. Nigeria with a population of about 120 million has a total of about 60 foundries. Only five out of the total foundries are commercial compared to about 9000 foundries in India [2]. Serious problems threatening the existence of these few foundries include energy and raw materials. The work partially addresses these problems particularly as it affect raw materials used in the making of cores. Organic binders are the best binders [3] and the cold setting binders save a lot of cost [4]. Unfortunately in this part of the world they are expensive and shortages are always often experienced thereby grounding production activities in the foundry [5]. To solve this problem an alternative to the common cold setting binder used in the foundries {urea formaldehyde furfuryl alcohol} was developed from manihot Esculenta and cement [5]. It is a cold setting binder system that does not require heating and it cures naturally [5, 6].

To determine the minimized cost of producing 1kg of core mixture using the developed binder system, the simplex method of linear programming was used; linear programming is a type of mathematical programming in which the mathematical functions describing a constrained problem are all linear [6-8]. Hadley had found that linear programming forms a natural technique for optimizing blending operation provided that the various quantities of interest blend linearly [8-10]. Linear programming have been used successfully in the blending of feeds at minimum cost. Powdered soap to meet specification for customer and petroleum products [7-8]. According to Lockyer most applications of linear programming fall into the following classes: product mix problems, blending problems, cutting problems, transportation problems and assignment problems. It can be used to maximize profit and to minimize cost [9].

Core is a very critical component of the mould; to produce a complex component with recess and holes a core must be used [10]. Sand cores are made of sand, binders and sometimes hardeners and catalyst [11, 12]. The fact that the core is an important component of the mould, an economical way for its production must therefore be sought for the survival of the few existing foundries in Nigeria [13].

Methodology

The procedure for the work was to get the composition for the core mixture, which included the composition of the developed binder system to be mixed with sand. The cost coefficients for the core mixture component were then determined, see Table 1.

Table1. Determination of cost coefficient for the core mixture Components

Components of the core mixture	Cost coef. C_j	Distance to and from source DT_j [Km]	Transport cost $C_T=4.5DT_j$		Cost of purchase CP_j		Weight of materials purchased w_j [Kg]	Total cost $cp_j+4.5DT_j$		Cost per unit weight $C_j = CP_j+4.5DT_j$	
			₦/km	\$/km	₦	\$		₦	\$	₦	\$/kg
Sand	C_1	50	225	1.73	1225	9.42	3000	1500	11.54	0.50	0.004
Manihot Esculenta Resin	C_2	20	90	0.69	1000	7.69	50	1090	8.39	21.80	0.17
Cement	C_3	20	90	0.69	1300	10.00	50	1390	10.69	27.80	0.21
Water (H_2O)	C_4	40	180	1.39	3000	23.08	2000	3180	24.46	1.59	0.01

Source of distance to and for source (Nigeria's shippers council, 2002)
Central Bank of Nigeria Foreign Exchange Rate is \$1.00 to ₦130

Cost parameter from the table above was

$$C_j = CP_j + 4.5DT_j \text{ and } j = 1, 2, \dots, n \quad (1)$$

The cost parameter were derived for each component of the mixture C_j by considering the following cost items

1. Cost of purchase at the source (CP_j)
2. Transportation cost ($C_T = 4.5DT_j$)

The next step was the setting up of the objective function.

Let

Sand = X_1 , manihot esculenta = X_2 , cement = X_3 , water = X_4

Minimize

$$C = 0.5X_1 + 21.8X_2 + 27.8X_3 + 1.59X_4 \quad (2)$$

This is a primal problem, where C is the cost of producing 1kg of core mixture.

Subject to

$$X_1 = 100 \quad (3)$$

$$X_2 = 5.0 \quad (4)$$

$$X_3 = 1.5 \quad (5)$$

$$X_4 = 2.0 \quad (6)$$

$$X_1, X_2, X_3, X_4 > 0$$

The constraints were established based on the developed resin binder and its composition in percentages.

The inverse or dual of the formulation above (equation 2) was then formulated. That is, making the formulation above into a maximizing problem by making a column for each limitation and a constraint row for each element in the objective function thus:

$$100X_1+5X_2+1.5X_3+2.0X_4 \quad (7)$$

Subject to: (Also slack variables were attached to the dual problem)

$$X_1+1S_1+0S_2+0S_3+0S_4=0.5 \quad (8)$$

$$X_2+0S_1+1S_2+0S_3+0S_4=21.8 \quad (9)$$

$$X_3+0S_1+0S_2+1S_3+0S_4=27.8 \quad (10)$$

$$X_4+0S_1+0S_2+0S_3+1S_4=1.59 \quad (11)$$

Equations 7 to 11 were then used in setting up the initial simplex table. The solution process continued up to the fifth simplex tableau, which gave an optimum solution with the composition row showing zeros and negative values through out. Table 2 shows the initial simplex tableau set up and table 3, shows the final simplex table. Figure 1 shows the simplex method flow chart.

Table 2. Initial Simplex Table

					Slack variables					
Row	X ₁	X ₂	X ₃	X ₄	S ₁	S ₂	S ₃	S ₄	Cost	TR (Test ratio)
S ₁	1(p.e)	0	0	0	1	0	0	0	0.50	0.5/1 = 0.5
S ₂	0	1	0	0	0	1	0	0	21.80	
S ₃	0	0	1	0	0	0	1	0	27.80	
S ₄	0	0	0	1	0	0	0	1	1.59	
Composition in %	100	5	1.5	2.0	0	0	0	0	0	

Table 3. Final Simplex Table

					Slack variables					
Row	X ₁	X ₂	X ₃	X ₄	S ₁	S ₂	S ₃	S ₄	Cost	
X ₁	1	0	0	0	1	0	0	0	0.5	
X ₂	0	1	0	0	0	1	0	0	21.80	
X ₃	0	0	1	0	0	0	1	0	27.80	
X ₄	0	0	0	1	0	0	0	1	1.59	
Composition %	0	0	0	0	-100	-5.0	-1.5	-2.0	-203.88	

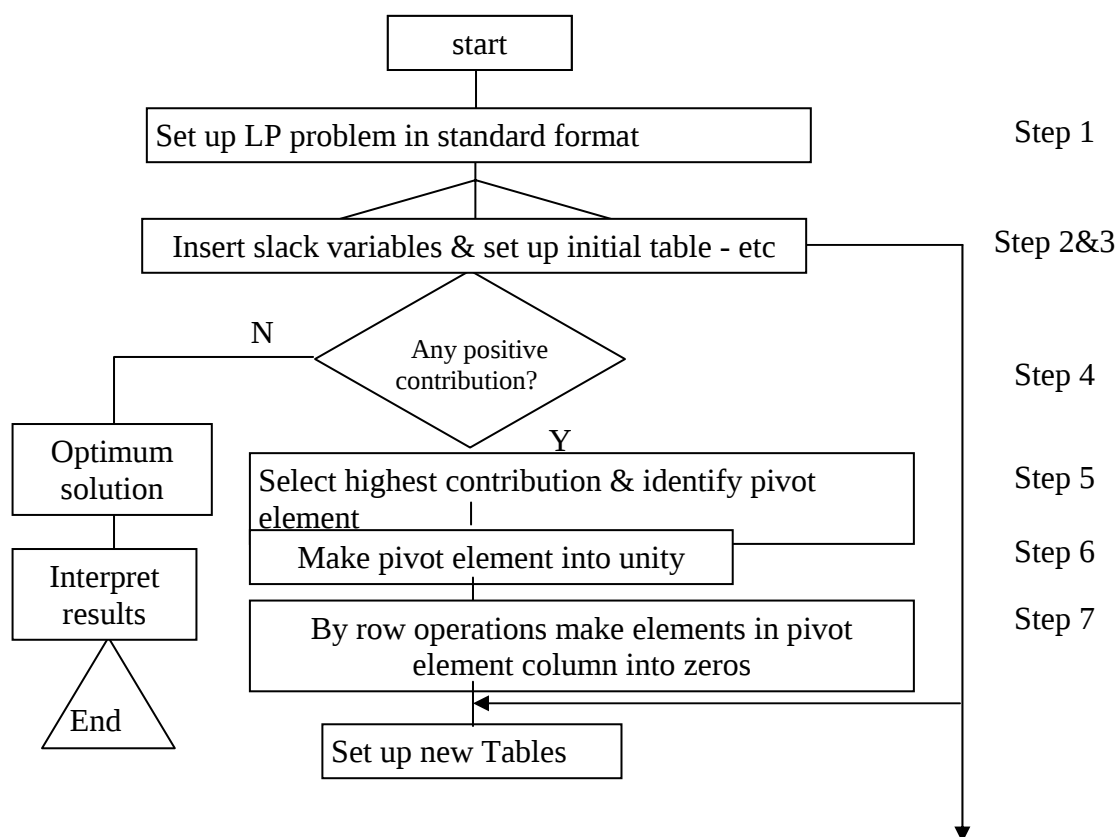


Figure 1. Simplex Method Flow Chart

Optimum Solution/Discussions

Compositional Percentages

The figures under the slack variable columns represent the compositional percentages for the preparation of the core mixture.

i.e

$S_1 = 100.0$	100% sand
$S_2 = 5.0$	5.0% manihot Esculenta Resin
$S_3 = 1.5$	1.5% cement
$S_4 = 2.0$	2.0% water

The compositional percentages correspond to the composition of the 1kg of the core mixture. All the percentages were based on the weight of the sand, this is a common practice in the foundry, see [11, 12].

Total cost of preparing 1kg of Core Mixture Using the Newly Developed Binder System

The figure at the bottom of the cost column represents total cost, i.e. 203.88 represents a cost of ₦203.88 (\$1.57), and it is the cost of preparing 1kg of core mixture with the above composition.

$$C = 0.5X_1 + 21.8X_2 + 27.8X_3 + 1.59X_4$$

Equation 12 is our cost minimization equation and our mathematical model for the minimal cost of production of our core mixture using the developed core binder system. Cost minimization is very important, one of the reasons for the development of this binder system as an alternative to urea formaldehyde furfuryl alcohol binder system is because of the high cost of the binder system. A similar quantity of core mixture prepared using urea formaldehyde furfuryl alcohol binder system would cause ₦650 (\$5) only. The newly developed binder system no doubt is better in terms of cost. With the composition known using equation 12 the minimized cost for the production of the core mixture can be confirmed. For instance with the above composition of 100% sand, 5.0% manihot Esculenta, 1.5% cement and 2% water the minimized cost will be as shown below;

$$100 \times 0.5 + 5.0 \times 21.8 + 1.5 \times 27.8 + 2.0 \times 1.59 = \text{₦}203.88 (\$1.57)$$

Valuation of Constraints

Rows titled X_1 , X_2 , X_3 , X_4 , indicate the cost of changing the limitation of X_1 to X_4 . If either of these limitations is changed by one unit, the total cost will change by 0.5, 21.80, 27.80, 1.59, i.e. these are the shadow prices. The constraints in this work were compositional constraints which their change would not only vary the cost but also vary or affect the core properties. The importance of working or keeping to the constraints is therefore very important. The properties of the cores produced from the core mixture strictly depend on the adherence to the constraints. The constraints are as stated in equations 3 to 6.

Conclusions

The LP simplex method model has been used in the determination of the minimized

cost for the production of 1kg of the core mixture using the newly developed core binder system.

The minimized cost of ₦203.88 (\$1.57) only was obtained which is cheaper than ₦650 (\$5) for the production of a similar quantity of core mixture using urea formaldehyde furfuryl alcohol binder system.

The work used compositional constraints; these constraints if changed would affect the total cost of producing the core mixture and invariably the core. The resultant properties of the cores produced from the core mixtures would also be varied.

Keeping to the constraints would therefore ensure quality production of cores from the developed binder system and would also minimize cost of production.

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