Key parameter values in a linear programming model for oil refinery production planning

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Abstract
The current paper covers the basic mechanisms of an impact of technological quality parameters on the solution of the model based on the linear programming approach, which is used for production planning at oil refineries. The basic aspects of the parametric analysis of a linear programming model are presented. It is shown that a linear programming model of an oil refinery contains quality parameters, which, if changed, can lead to a discontinuity of the first kind of the maximum value of the objective function. The importance of determining the values of the above-mentioned parameters in a linear programming model (the so-called key values of the parameters) is demonstrated. It is shown that the knowledge of key parameter values makes possible to determine the opportunities for significant increase of the refinery’s profit through small changes of the parameter in the neighbourhood of its key value. It means that a small impact on the production process by slightly changing the corresponding process variable can provide a significant increase in the economic efficiency of the refinery. Approaches to the optimal choice of parameter values for linear programming models for increasing economic efficiency of an oil refinery are also considered.

Keywords
Linear programming model; Production planning; Oil refinery

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Introduction

Currently, the vast majority of oil refineries calculate their production plans using specialized planning optimization systems. The key vendors of such solutions are Aspen Tech (PIMS), Honeywell (RPMS), and Haverly (GRTMPC). These systems operate based on the mathematical model of production incorporated in them and use methods of linear programming (LP) to find an optimal solution (or plan) according to the maximum marginal profit criterion. The effectiveness of LP systems in calculating refinery production plans is undoubtedly very high. At the same time, construction of an adequate refinery production model in a LP system is rather complicated. Errors and inaccuracies in LP models lead to significant economic losses. That is why at present much attention is focused on methods and approaches to the construction of an LP model of an oil refinery. A lot of discussion in the literature is devoted to the problem of improving the planning accuracy by increasing the complexity of the LP model [1-3]. However, specific approaches to optimize a refinery production plan based on LP systems are scantily described.

Refinery models, which are implemented in LP systems, contain a large number of various parameters. In general, these parameters can be subdivided into two principal categories: economic (or market) and technological ones. Economic parameters include prices and constraints on supplies of raw materials and product marketing. The technological parameters involve constraints on the yields of the process units and their capacities, as well as the qualities of semi-finished and marketable products. Technological parameters also include norms of consumption of auxiliary raw materials, additives, energy, reagents and catalysts. The impact of economic parameters is described in the literature in more detail. At the same time, the effect of production parameters is considered much less often. This is due to the complexity of refinery models (a LP matrix typically consists of several thousand rows and columns), which complicates the analysis and development of analytical correlations or recommendations that could help production planning specialists in their routine work.

At present, the prevailing opinion is that the main factor of successful optimization of a production plan is maximum account in LP models of all types of technological and logistic variants for refinery units. For this purpose, various models, including nonlinear ones, have been introduced [4-6] together with mathematical algorithms in order to solve nonlinear LP [7-8]. After all the specific characteristics of an oil refinery have been incorporated into a LP
model, the optimal planning system generates an optimal production plan. On the other hand, using rigorous process models for refinery planning imposes unnecessary complications of the problem because such models increase the solution time and often conceal critical issues and parameters for increasing profit [9-11]. Another problem is connected with constraints of the LP system solver. This is determined by the fact that, as soon as the size of the model (i.e., the number of variables and constraints) becomes too large, there occur errors in the model solution (the so-called solution divergence), which are caused by some limitations of the mathematical algorithms used in the solver of LP systems. Another drawback of LP models is the problem caused by accounting for logistical constraints and phased operation settings (for example, in the framework of a LP model, two different types of products are produced at the same unit simultaneously rather than successively, as it is in fact).

The above-mentioned factors lead to the fact that in most practical cases LP models do not incorporate all the variety of possible situations, but only present some basic options and parameters of an oil refinery. At the same time, to understand which parameter value provides the optimal effect, in some cases it is necessary to change the corresponding parameter value in the LP model and restart the solution procedure. The latter means that, in fact, the optimal plan obtained through the solution of the LP model turns out not to be the best in the global sense, but only within those parameter values, which are included in the model. Thus, true optimization is very difficult [12]. If parameters of LP model are changed, the optimal production plan can change as well. Thus, it can be concluded from what was mentioned above that, in order to optimize the production plan, first the parameters of LP model must be "optimized", which implies the need for its parametric analysis. Given the fact that a LP model of a realistic oil refinery contains a huge number of parameters, and taking into account the constantly changing of the external conditions [13-14], a detailed analysis of the model is quite difficult and requires a considerable amount of time, which, in most practical cases, is insufficient or absent. In this regard, experience and qualification of specialists that work with the model is the issue of vital importance. The required qualification is obtained through long (in most cases, years long) experience of working with the LP model. As a result, such specialists are already familiar with the LP model "bottlenecks", i.e., parameters’ values that may significantly affect the solution, saving time expenditures on the total parametric analysis. Thus, the effectiveness of optimal production plans largely depends on the human factor, and therefore continuous perfection of planning skills of specialists is also of high
importance.

At the same time, approaches to a parametric analysis of refinery LP models that are described in the literature are not sufficient. The latter is largely explained by significant differences between LP models of various configurations of refineries. As a result, at best case scenario, specialists, who have not worked with optimal planning issues long enough, make use of empirical experience of their colleagues. Thus, economic losses are inevitable and significant. That is why the purpose of this paper is to study the mechanisms of parameter influence on the optimal production plan of an oil refinery. Special attention is paid to determine such parameters, whose changes, could qualitatively vary the value of the objective function of an oil refinery, which, at the same time, is the planned margin profit of the oil refinery.

**Material and method**

**Production planning process**

Typical scheme of production planning process at an oil refinery is presented in the Figure 1.

The information which is necessary for production planning is collected in LP system (PIMS, RPMS or GRTMPC) from different divisions of refinery. Supply and delivery division provides volume constraints and prices of raw material and finished products which
can be discharged/loaded in planning period. Technological division is responsible for process flows qualities data, constraints for yields and capacities of process units, blending recipes, as well as norms of utilities, additives, reagents and catalysts consumption. Production division provides technological flows scheme. Once all necessary data is loaded in LP system or, as it is customarily said, the refinery LP model is formed in LP system, after that the optimal plan search process is started.

Specific characteristics of an LP model of oil refinery

Mathematically the task of optimal production planning of an oil refinery can be written as follows, equation (1):

\[
L = \sum_{j=1}^{m} c_{j}x_{j} - \sum_{j=m+1}^{n} d_{j}x_{j} \rightarrow \max,
\]

Where: \( L \) – objective function which has an economic meaning of marginal profit; \( x_{j} (j = 1, ..., m) \) – variables of the sales (products); \( x_{j} (j = m+1, ..., n) \) – variables of purchases (crude oil, energy consumption, additives, reagents, etc.); \( c_{j} \) – sale prices ; \( d_{j} \) – purchase prices;

\[
\sum_{j=1}^{n} a_{ij}x_{j} + \sum_{j=n+1}^{p} a_{ij}x_{j} \leq (\geq) b_{j},
\]

\[x_{j} \geq 0, \ j = 1, ..., p.\]

Where: \( x_{j} (j = n + 1, ..., p) \) – variables of internal flows (semi-finished products, consumption of energy, additives, reagents, etc.), \( a_{ij} \) – coefficients of the matrix constraint. System (1) – (3) is also defined as a LP model.

If the second group of summands \( x_{j} (j = n + 1, ..., p) \) is omitted in equation (2), LP model (1) – (3) reduces to a classical form. Qualitative aspects of the effect of parameters \( a_{ij} (j = 1, ..., n), c_{j}, d_{j} \) and \( b_{j} \), upon its solution have been studied in detail in the literature (e.g., [15]). The most interesting case of the LP model solution behaviour is observed in the situation of changing parameter \( a_{ij} \) (or \( c_{j} \)) which leads to an abrupt discontinuity in the optimal plan solution. In this case, the dependence of optimal values of a part of variables \( x_{j} \) on the parameter exhibits a discontinuity of the first kind [15]. At the same time, the optimal value of the objective function remains continuous.

The specific character of a LP model of an oil refinery is in that, apart from variables \( x_{j} (j = 1, ..., n) \), which are present in equation (1) and pertains to purchases/sales, in
inequalities (2) there are additional variables $x_j$ ($j = n + 1, \ldots, p$), which are absent in an explicit form in expression (1) for the objective function. These variables are required for describing the volumes or other characteristics of the semi-finished products in the model. The analysis of the effects determined by the parameters $a_{ij}$ ($j = n + 1, \ldots, p$) on a LP model is not, as a rule, given any special attention. At the same time, it will be further shown that the presence of parameters $a_{ij}$ ($j = n + 1, \ldots, p$) in inequalities (2) can lead to the appearance of qualitatively new parametric impact in the solution of the LP model of an oil refinery, in contrast to its classical form.

Parameters $a_{ij}$ ($j = n + 1, \ldots, p$) at an oil refinery can have the following meaning. Firstly, they determine proportions (material balance) of yields of oil products at the process units. Secondly, $a_{ij}$ ($j = n + 1, \ldots, p$) are used to set norms of consumption of energy, reagents, catalysts and additives at the process units. Thirdly, they determine qualitative characteristics of flows (semi-finished products), for example, density, viscosity etc. It can be readily shown that in the first and second cases parameters $a_{ij}$ ($j = n + 1, \ldots, p$) determine relations between $x_j$ ($j = 1, \ldots, n$) and $x_j$ ($j = n + 1, \ldots, p$), which are defined using linear equations. As a result, any of variables $x_j$ ($j = 1, \ldots, n$) can be written in equation (1) using a linear combination of variables $x_j$ ($j = n + 1, \ldots, p$). It means that in this case parameters $a_{ij}$ ($j = n + 1, \ldots, p$) will have the same qualitative effect on the optimal solution of the LP problem as $a_{ij}$ ($j = 1, \ldots, n$).

In the third case, constraints connected with quality are usually written in the form of inequalities. Consider possible parametric effects in this case in more detail. Let marketable product $x_0$, which is a result of blending of $k$ components, have to comply with certain minimal (maximal) values of quality indexes ($\lambda_0^{(1)}, \lambda_0^{(2)}, \ldots, \lambda_0^{(s)}$) according to a certain standard. Depending on the kind of marketable product, the value of $s$ can vary from several units to two tens. The following parameters are used as quality indices: density, sulphur content, viscosity, aromatics, temperature numbers, octane numbers etc. Constraints (2) for $x_0$ will be written as follows, Eq. (4):

$$
\lambda_0^{(1)} x_0 - \lambda_1^{(1)} x_1 - \ldots - \lambda_k^{(1)} x_k \leq (\geq) 0,
$$

$$
\lambda_0^{(2)} x_0 - \lambda_1^{(2)} x_1 - \ldots - \lambda_k^{(2)} x_k \leq (\geq) 0,
$$

$$
\ldots
$$

$$
\lambda_0^{(s)} x_0 - \lambda_1^{(s)} x_1 - \ldots - \lambda_k^{(s)} x_k \leq (\geq) 0.
$$

To simplify the analysis of the effect of quality indices $\lambda$ on the optimal solution of the LP problem, and keeping in mind equality $x_0 = x_1 + x_2 + \ldots + x_k$, system of inequalities (4)
can be rewritten as, Eq. (5):

\[
\begin{align*}
\lambda_0^{(1)} - \lambda_1^{(1)} &\leq x_1 + \cdots + \lambda_0^{(k)} - \lambda_k^{(k)} \\
\lambda_0^{(2)} - \lambda_1^{(2)} &\leq x_1 + \cdots + \lambda_0^{(k)} - \lambda_k^{(k)} \\
\vdots \\
\lambda_0^{(s)} - \lambda_1^{(s)} &\leq x_1 + \cdots + \lambda_0^{(k)} - \lambda_k^{(k)} \\
\end{align*}
\]

The geometrical meaning of constraints (5) can be formulated as follows. The left sides of inequalities (5) define in a \(k\)-dimensional space multidimensional “planes”, which are defined by the vectors, Eq. (6):

\[
\begin{align*}
N^{(1)} &= \gamma^{(1)}(\lambda_0^{(1)} - \lambda_1^{(1)}, \lambda_0^{(1)} - \lambda_2^{(1)}, \ldots, \lambda_0^{(1)} - \lambda_k^{(1)}) \\
N^{(2)} &= \gamma^{(2)}(\lambda_0^{(2)} - \lambda_1^{(2)}, \lambda_0^{(2)} - \lambda_2^{(2)}, \ldots, \lambda_0^{(2)} - \lambda_k^{(2)}) \\
\vdots \\
N^{(s)} &= \gamma^{(s)}(\lambda_0^{(s)} - \lambda_1^{(s)}, \lambda_0^{(s)} - \lambda_2^{(s)}, \ldots, \lambda_0^{(s)} - \lambda_k^{(s)}) \\
\end{align*}
\]

Here, an additional coefficient \(\gamma\) is introduced, which is equal to “–1” or “+1”, depending on the sign “≤” or “≥” in the right side of (5).

Let’s also mention here the condition of collinearity of vectors \(N^{(\alpha)}\) and \(N^{(\beta)}\) \((\alpha \in (1,s), \beta \in (1,s))\). Obviously, this can happen if the following relation holds, Eq. (7):

\[
\frac{\lambda_0^{(\alpha)} - \lambda_1^{(\alpha)}}{\lambda_0^{(\beta)} - \lambda_1^{(\beta)}} = \ldots = \frac{\lambda_0^{(\alpha)} - \lambda_k^{(\alpha)}}{\lambda_0^{(\beta)} - \lambda_k^{(\beta)}}.
\]

**Results and discussion**

**Parametric analysis of the LP model with one quality parameter**

Consider a simplified example of the processing of raw material at a process unit of maximal capacity \(b\) (see Figure 2).
The gasoline component $x_3$ with octane number $\lambda$ can be directed to the production of gasolines $x_{01}$ and $x_{02}$ of minimum acceptable octane numbers $\lambda_{01}$ and $\lambda_{02}$ ($\lambda_{01} < \lambda_{02}$) and prices $c_1$ and $c_2$ ($c_1 < c_2$), respectively. The solution of LP model (1) – (3) for the scheme shown in Figure 2 is presented in Figure 3a-c.

It is obvious from the Figure 3 that, apart from a jump of the optimal plan of $x_{01}$ and $x_{02}$ for parameter values: $\lambda = \lambda_{01}$, $\lambda = \lambda_{02}$ a discontinuity of the first kind of the optimal value of objective function $L_{\max}$ is also observed. In what follows, such parameter values will
be called “key values”. The geometrical meaning of the appearance of a discontinuity of function $L_{\text{max}}(\lambda)$ is depicted in Figure 4.

For $\lambda < \lambda_{01}$, the tolerance region for $x_{01}$ and $x_{02}$ degenerates into point (0,0). For $\lambda = \lambda_{01}$, a discontinuous increase of the tolerance region to a line segment, AD, is observed. When the octane number of the component reaches the value of $\lambda = \lambda_{02}$, there is another jump in the increase of the tolerance region for $x_{01}$ and $x_{02}$, which transforms from segment AD to square ABCD.

**Parametric analysis of the LP model with several quality parameters**

In the above example, it is shown that, when the quality index value of the gasoline component coincides with the minimum (maximum) admissible quality of the marketable product, this parameter value of the component can be a key value. Then a question arises whether the key parameter value of a gasoline component can differ from the minimum (maximum) admissible quality of the marketable product. To answer the question, in accordance with what was said in the above paragraph, let us try to find such a value, for which a discontinuous change of the tolerance region of the variables is possible. Consider the geometrical meaning of the system of vectors (6). Corresponding «multidimensional» planes bound the tolerance region for $x_1, \ldots, x_k$. Further, a situation similar to Figure 4 must be considered where discontinuous changes in the size of the tolerance region for $x_1, \ldots, x_k$ are possible if the parameters of $\lambda$ are changed. Obviously, this can happen when there are two
collinear and oppositely directed vectors: \( \exists \alpha \in (1, s) \), \( \exists \beta \in (1, s) \): \( N^{(\alpha)} \uparrow \downarrow N^{(\beta)} \). The latter implies that the condition (7) is fulfilled.

It should be being noted that equation (7) is not a sufficient condition for the existence of a discontinuity in the relation between the maximal value of the objective function and the parameter. This is determined by the fact that, if the sale of product \( x_0 \) is not the optimal variant, then the parametric effect upon the LP system will have no effect upon the solution of the problem. It also should be noted that in comparison with example shown in Figures 2-4 where it was discussed about key parameter value, in this case key proportions of the values of several parameters should be considered.

Situations with discontinuities of the first kind in the objective function, when \( N^{(\alpha)} \uparrow \downarrow N^{(\beta)} \), are clearly manifested in the optimal production planning of a realistic oil refinery (Petrotel-LUKOIL SA) using program RPMS (Honeywell). Constraint matrix (2) of this oil refinery has the dimensions of (2300×3600). An example of a complete solution for Petrotel-LUKOIL SA is not presented here, as the large amount of comments and explanations required for the formulation and solution of the problem will conceal the main mechanisms of formation of a discontinuity of the maximum objective function. That is why a simplified example will be used, which, nevertheless, has a direct bearing on the real situation at the oil refinery.

Consider the optimization of production of marketable gasolines \( x_{01}, x_{02} \) (see Figure 5).

Here \( \lambda_{01} \) and \( \lambda_{02} \) (\( \lambda_{01} < \lambda_{02} \)) are minimum admissible octane numbers, \( a_{02} \) is maximum aromatic content for the second gasoline; for the first gasoline there are no constraints on aromatics \( a_{01} \). The marketable gasolines with margin profits \( c_1, c_2 \) (including the costs) are results of blending the components from the isomerization and catalytic
reforming units, which produce the same amounts of gasoline components, \( b \). Let the products from the units have octane numbers \( \lambda_1, \lambda_2 \) and aromatic contents \( a_1, a_2 \).

In Figures 6 and 7 is shown the result of numerical calculation using RPMS software for Figure 5 and Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \min (\lambda_{01}) ) RON</th>
<th>( \min (\lambda_{02}) ) RON</th>
<th>( \min (a_{01}) ) %</th>
<th>( \min (a_{02}) ) %</th>
<th>( \lambda_1 ) RON</th>
<th>( a_1 ) %</th>
<th>( a_2 ) %</th>
<th>( b ) th. tons/day</th>
<th>( c_1 ) $/ton</th>
<th>( c_2 ) $/ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>92</td>
<td>95</td>
<td>0</td>
<td>42</td>
<td>85</td>
<td>0</td>
<td>63</td>
<td>1</td>
<td>450</td>
<td>500</td>
</tr>
</tbody>
</table>

Figure 6. Optimal value of objective function \( L_{\max} \) versus parameter \( \lambda_2 \)

Figure 7. Optimal plan \( x_{01}, x_{02} \) versus parameter \( \lambda_2 \)

The figure 6 shows a discontinuity of \( L_{\max}(\lambda_2) \) if \( \lambda_2 = 100 \). It is easy to calculate that in this case for \( \lambda_{02}, \lambda_1, a_{02}, a_1, \lambda_2, a_2 \) the proportion (7) is satisfied.

The important consequence of this example is as follows. If it is known that catalytic reforming unit (see. Figure 5) can produce gasoline component with octane number value \( \lambda_2 \)
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in the range, for example, from 99.0 till 100.5, and specialist in production planning indicates in RPMS software the average of possible \( \lambda_2 \) values as 99.75, then the economic losses of calculated in such a way optimal plan (see Figure 6) will be 75 th. $/day. In turn, knowledge of key parameter values allows the production planner to indicate the «optimal» value of parameter as \( \lambda_2 = 100 \) and helps to avoid such an economic loss.

Conclusions

The specific properties of the parametric analysis of a LP model of an oil refinery have been considered. It is shown that a LP model of an oil refinery contains quality parameters, which, if changed, can lead to a discontinuity of the first kind of the maximum value of the objective function. The importance of determining the values of the above-mentioned parameters in a LP model (the so-called key values of the parameters) is demonstrated. It is shown that knowledge of key parameter values makes it possible to determine possible ways of significantly increasing the profit margin of an oil refinery through small changes of the parameter in the neighbourhood of its key value. It means that a small impact on the production process by slightly changing the corresponding process variable can provide a significant increase in the economic efficiency of the refinery.

Determining key parameter values in a LP system of a real oil refinery, described by thousands of equations, is not trivial. What makes this task still more difficult is that the software currently used for solving optimization problems (PIMS (Aspen Tech), RPMS (Honeywell), Haverly (GRTMPC)) does not always look for the solution of the LP system in the interval of all the possible parameter values, but only for some of their values. That is why a parametric analysis of a LP model in that case requires numerous restarts of the program for different parameter values. As a complex system comprises quite a large number of such parameters, the quality of optimal planning of an enterprise depends, to a large extent, on the qualifications and experience of the planning personnel.

In practice, the parametric analysis of a LP model solution in most cases is carried out with the direct interaction of the planning and technological departments. Proposals to improve the production efficiency result from the parametric analysis of the solutions of LP models, and are submitted by the planning department to the technological one. If the technological department confirms the possibility of such changes (by changing the severity
of the regime in the process unit), then the adjustments are done in the LP model. In most cases, a change of a parameter can affect other parameters associated with it, which should also be considered in the LP model (changes in the quality of the product are directly accompanied by changes in the yields of the unit where the product is produced).

References


