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A new design for a robust fractional Smith predictor controller based fractional model

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Abstract

The present paper deals with the fractional controller synthesis method based on the Smith Predictor (SP) scheme using the fractional model, where the obtained feedback control system satisfies both time and frequency domains specifications. Both fractional controller and model parameters are designed through solving two proposed constraint optimization problems and using the same meta-heuristic algorithm, called Particle Swarm Optimization (PSO). The proposed method is used to control some actual systems where the dynamic behaviour of each one is modelled by the mismatch model with dead time. The obtained simulation results are compared in time and frequency domains with those given by the conventional integer and fractional SP controllers to validate the effectiveness of the proposed method.

Keywords

Fractional calculus; Dead time; Smith Predictor; Fractional model; Fractional PID controllers; Optimization; PSO Algorithm; Identification

Introduction

Systems that possess a dead time are hard to be controlled by the classic controllers such as the proportional integral derivative (PID) controller [1]. However, they could be controlled using the Smith Predictor (SP) scheme, which is commonly used for this type of systems [2, 3]. This strategy control has been widely studied in recent research papers with different propositions leading to one goal [4]: an optimal controller based upon an optimal model obtained in two steps, which are (1) the modelling step and (2) the controller synthesis step [5]. In order to ensure a low tracking error index, a low-cost control, and a less sensitivity to both sensor noises and load disturbance signals [6, 7].

It is well known that the development of tuning rules, for the process with dead time, requires a perfect model that provides an ideal mathematical model with adequate details [8]. This model should provide a precise description of the real dynamic of the process to be controlled. Unfortunately, a perfect mathematical model is rarely available in practice due to the strong nonlinear dynamic of the process [9, 10]. To provide an exact description of the system, the process should be reduced in a suitable template such as a First-Order Plus Time Delay (FOPTD) model [8, 11] or a Second-Order Plus Time Delay (SOPTD) model [8, 12]. Various tuning rule methods based upon these models have been developed to approximate the dynamic of the process and guarantee the good robustness margin of the feedback control system [8, 13]. Among them, Zhuang and Atherton [14] proposed a tuning rule method for integer order PID controllers to handle FOPTD model, which is rather a poor approximation for higher-order processes [15]. Palmor [16] developed an auto-tuning algorithm for the Smith dead time compensator using a FOPTD model for stable plants. This algorithm estimates two points on the process of the Nyquist curve via automatic generation of the controlled limit cycles. The obtained points are then used in a least square procedure to estimate the model parameters, in which a primary integer order controller has been designed through the empirical Zeigler-Nichols method. Hang [17] proposed a method for auto-tuning and self-timing the modified SP using the relay feedback strategy. On the other hand, Wang [11] proposed an analytical way to design series of FOPTD models, in which the obtained controller is designed by only one of them. Kaya [18] suggested three types of integer order models, which are: a stable SOPTD, an integrating FOPTD, and an unstable FOPTD. Unfortunately, the previous integer models can generate errors in the estimation step of the model parameters, due to a large dead time delay and a higher order process. Baiyu [19] used the Radial Basis Function Neural Network (RBFNN) method to find FOPTD parameters for a nonlinear process [20, 21]. However, this method did not produce an analytical expression for the model. As a result, the controller parameters are not easy to design using the previous strategies [22, 23]. Notice that, the fractional order carried on the Laplace operator can be approximated into the usual integer transfer function that contains the infinite poles and zeros. Therefore, many advanced control strategies benefit to the previous property, in which the obtained robust fractional controller may be synthesized with less unknown parameters [24]. The obtained controller should satisfy several imposed specifications when the model parameters change in a wide range. Consequently, the size of the optimization problem becomes considerably reduced ensuring the success of the optimization tool.

The aim of this work is to enhance the performances of the closed loop SP scheme for systems with dead time. To achieve this goal, a new fractional-order model has been proposed. Also, a synthesis method based on SP scheme and an optimization problem with reduced constraints to design the proposed robust SP fractional controller. This proposed strategy ensures a good rejection dynamic, a good set-point tracking dynamic, and a good attenuation dynamic of the sensor noises for two different plants.

Material and method

In this section, we will present methods which are used in the modelling and the controller synthesis steps to obtain the proposed controller. But for the sake of clarity, let us define the basic notations of the fractional calculus and of the SP method [25].

Smith Predictor

The scheme proposed by Smith for the dead time compensation is shown in figure 1.

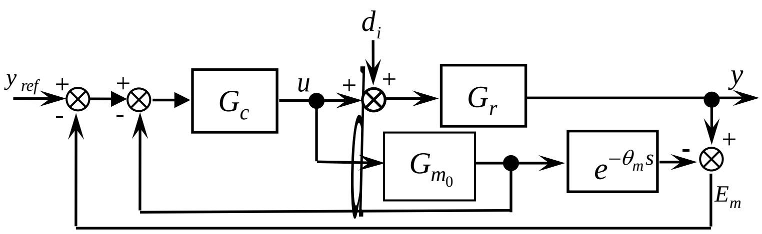

Figure 1. Smith Predictor scheme

Where: y, u, y_{ref} and d_i denote, respectively, the process output, the control signal, the set-point reference, and the plant input disturbance. Moreover, G_r is the actual process, $G_m = G_{m_0} e^{-\theta_m s}$ is the proposed fractional model, which will be determined later by an adequate optimization tool. G_c is the proposed fractional controller, which will be designed through the previous fractional model. Finally, G_{m0} is the free dead time part of the proposed model G_m.

In this paper, the SP synthesis controller has two phases (*i*) the modelling step, and (*ii*) the controller synthesis step. In the modelling step, the main goal is to find an efficient fractional model that perfectly fits the frequency response of the actual process. In addition, the main goal of the controller synthesis step is to design a robust fractional controller that satisfies several specifications.

Fractional calculus

The fractional calculus is the subject of interest for more than three centuries. The number of applications where the fractional calculus has been used is growing rapidly. The obtained fractional mathematical model can describe a real object with greater accuracy than conventional integer model. Fractional calculus is a generalization of the integration and differentiation to the fractional order fundamental operator ${}_aD_t^\alpha$, where a and t are the limits and $\alpha \in \Re$ is the order of the operation. Among many definitions, two are commonly used for the generalized fractional integral-derivative operations, which are the Grunwald-Letnikov (GL) and the Riemann-Liouville (RL) definitions.

The GL definition is given by Eq. (1), [25]:

$$ {}_aD_t^\alpha f(t) := \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[t-a/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{1}$$

Where: $\left[t-a/h\right]$ means the integer part.

The RL definition is given by the following Eq.(2), [25]:

$$ {}_aD_t^\alpha f(t) := \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\varphi)}{(t-\varphi)^{\alpha-n+1}} d\varphi \tag{2}$$

Where: $n-1<\alpha<n,$ and $\Gamma(x)$ is the Euler's *Gamma* function.

For convenience, the Laplace domain is commonly used to describe the fractional integral-derivative operation. According to the RL definition, the Laplace transform of equation (2), under zero initial condition and $0<\alpha<1$, is given by equation (3):

$$ L[\ {}_aD_t^\alpha f(t)] = s^\alpha . F(s) \tag{3}$$

Notice that, the fractional-order transfer function (FOTF) of powers of α_n requires approximating its fractional parts into usual integer transfer function with a similar behaviour. This goal is achieved by using the Oustaloup method [24] where s^α can be approximated in the specified frequency range $[\omega_b, \omega_d]$ with the choice of the integer number N, Eq. (4):

$$s^{\alpha} = C_{\alpha} \prod_{k=-N}^{k=N} \frac{s+\omega'_k}{s+\omega_k} \tag{4}$$

According to equation (4), zeros, poles, and gain are respectively given by Eq. (5-7):

$$\omega'_k = \omega_d \left(\frac{\omega_b}{\omega_d}\right)^{\frac{k+N+0.5(1-\alpha)}{2N+1}} \tag{5}$$

$$\omega_k = \omega_d \left(\frac{\omega_b}{\omega_d}\right)^{\frac{k+N+0.5(1+\alpha)}{2N+1}} \tag{6}$$

$$C_{\alpha} = \left(\frac{\omega_b}{\omega_d}\right)^{\frac{\alpha}{2}} \prod_{k=-N}^{N} \left(\frac{\omega_k}{\omega'_k}\right) \tag{7}$$

The implementation problem of s^{α} (where $\alpha > 1$) is solved by considering $s^{\alpha} = s^{n+\psi}$ where s^n is the integer part of s^{α} and s^{ψ} (where $0 < \psi < 1$) is determined by the Oustaloup method [24] using equations (4) to (7).

On the proposed fractional model

In the modelling step, the fractional model parameters are determined from solving the proposed optimization problem using the particle swarm optimization (PSO) algorithm. The cost function is formulated as the Least Mean Square Error LMSE criterion, which is given by the discrepancy value, at each logarithmic space frequency, between both magnitudes of the actual process and the proposed fractional model. Furthermore, the inequality constraints are given through the absolute discrepancy value, at each logarithmic space frequency, between both phases of the actual process and the proposed fractional model. Hence, the transfer function of the proposed fractional model is Eq. (8-9):

$$G_m(s) = G_{m_0}(s).e^{-\theta_m s} = \left(\frac{K_m}{1+T_{m_1} s^{\alpha_m}}\right)\left(\frac{e^{-\theta_m s}}{1+T_{m_2} s}\right) \tag{8}$$

$$x_m = \left[K_m, T_{m_1}, T_{m_2}, \alpha_m, \theta_m\right]^T \tag{9}$$

Where: the model parameters are given by the vector $x_m \in \Re^5$ (see equation (9)), which are: K_m gain, T_{m_1}, T_{m_2} - time constants of first order transfer function, α_m - the order of the fractional system, and θ_m - the dead time value.

Notice that, the above fractional model (see equation (8)) combines two transfer functions. The first one, given in the right, presents the integer FOPTD model that is commonly used in the modelling step of the standard SP design method. However, the second transfer function, given in the left, presents the stable multi-lag fractional model, which will be used later in the controller synthesis step. Moreover, the proposed fractional model parameters are usually determined by resolving the following optimization problem:

$$\begin{aligned} &\min_{x_m} \|E_m\|_\infty = \min_{x_m} \|G_r(s_i) - G_m(s_i, x_m)\|_\infty \\ &\quad \text{subject to: } x_{\min} \le x_m \le x_{\max} \end{aligned} \tag{10}$$

According to previous works, several methods have been proposed to reformulate the optimization problem given in equation (10). Among them, the chosen optimization problem is defined as follows Eq. (11):

$$\begin{aligned} &\min_{x_m} J_m(x_m) = \min_{x_m} \frac{1}{n}\sum_{i=1}^{n} \left|E_m(s_i, x_m)\right|^2_{s_i = j\omega_i} \\ &\text{subject to: } \begin{cases} h_{m_i}(x_m) = \left|\arg\left(E_{m_i}(s_i, x_m)\right)_{s_i = j\omega_i}\right| - \varepsilon_m \le 0 \\ x_{\min} \le x_m \le x_{\max} \end{cases} \end{aligned} \tag{11}$$

Where: J_m denotes the cost function to be minimized, n is the number of frequencies chosen from the frequency range $\omega \in [\omega_{low}, \omega_{high}]$, ε_m is the positive scalar which presents the minimum distance allowed between both phases of the actual process and the proposed fractional model, $x_{\min}$ and $x_{\max}$ are, respectively, the lower and upper bounds that limit the design vector x_m, h_{m_i}, where $i = 1, \cdots, n$ are the inequality constraints presenting the absolute error between both phases of the actual process and the proposed model.

Knowing that, the optimal solution of the optimization problem given by equation (11) is difficult due to the high number of the inequality constraints. To resolve this difficulty, the optimization problem will be transformed into the bounded constraints optimization using the same idea proposed in [23]. Afterward, the fractional model parameters are easily determined from its optimal solution using the PSO algorithm.

Synthesis Controller Step

In synthesis controller step, the proposed fractional controller is designed using the

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similar way proposed by Wang in [11]. Therefore, the free dead time part of the proposed model G_{m_0} can be decomposed into two transfer functions such as, Eq. (12):

$$G_{m_0} = g_{m_{01}} \times g_{m_{02}} = \left(\frac{K_m}{1 + T_{m_1} s^{\alpha_m}} \right) \times \left(\frac{1}{1 + T_{m_2} s} \right) \tag{12}$$

Where: $g_{m_{01}}$ and $g_{m_{02}}$ are, respectively, the fractional and integer free dead time parts of the proposed fractional model. Hence, the modified mismatch SP scheme is shown in figure 2 [11]:

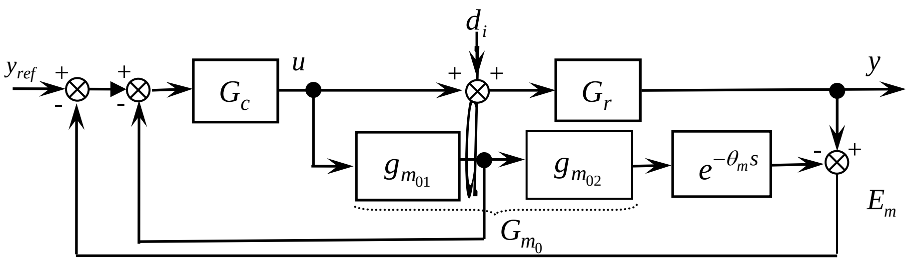

Figure 2. Smith Predictor

It can be simplified as the following modified mismatch SP schemes, which are given by both figures 3 and 4 [11].

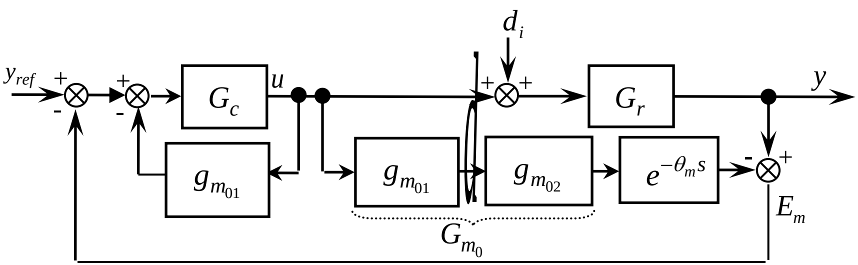

Figure 3. Modified mismatch SP (first version)

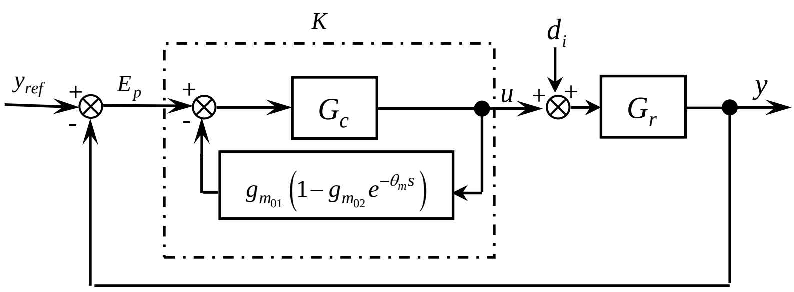

Figure 4. Modified mismatch SP (second version)

Where: E_p denotes the tracking error between the process output and the set-point reference.

Let us consider now x_c the controller parameter vector that defines the transfer function of the proposed fractional controller $K(s)$, which is defined by Eq. (13):

$$K(s,x_c)=\frac{G_c(s,x_c)}{1+G_c(s,x_c)\left[g_{m_{01}}(s)-g_{m_{01}}(s).g_{m_{02}}(s).e^{-\theta_m s}\right]} \tag{13}$$

According to figure 4, the general scheme of the closed-loop system can be presented.

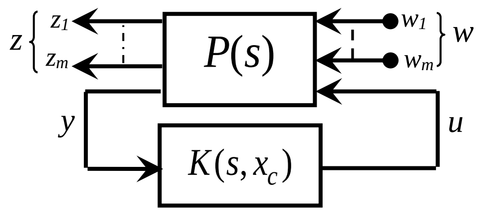

Figure 5. Block diagram of the feedback control system

Where: $P(s)$ is the generalized plant given by Eq. (14):

$$\begin{bmatrix} z \\ y \end{bmatrix} = P(s)\begin{bmatrix} w \\ u \end{bmatrix},\ u = K(s,x_c).y \tag{14}$$

As depicted in figure 5, the closed-loop system has m exogenous inputs, which are presented by the vector $w=\left[w_1,w_2,...,w_m\right]^T$ including various exogenous inputs such as sensor noises, plant input disturbances d_i, plant output disturbances d_o, and set point references y_{ref}. In addition, it has also m exogenous outputs that are presented by the vector $z=\left[z_1,z_2,...,z_m\right]^T$ where all imposed characteristics are considered. The proposed fractional controller $K(s,x_c)$ utilizes the measured output vector y to elaborate the control vector u, which modifies the natural behaviour of the process $P(s)$ (see equation (14)). The obtained controller should satisfy some imposed specifications such as, a good tracking dynamic of set point references, a good attenuation of all load disturbances and sensor noises. These requirements are formulated as the following optimization problem [5]:

$$\begin{aligned} &\min_{x_c} J_c(x_c) = \min_{x_c}\left\|J_{w\to z}\left(s,x_c\right)\right\|_\infty \\ &\quad \text{subject to: } x_{c_{\min}} \le x_c \le x_{c_{\max}} \end{aligned} \tag{15}$$

Where: $J_{w\to z}$ denotes the closed-loop transfer matrix between both exogenous input w and output z vector.

Proposed Fractional-order Controller

The proposed robust fractional controller is given by the following fractional structure,

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Eq. (16):

$$G_c(s,x_c)=G_{c_1}(x_c).G_{c_2}(s,x_c).G_{c_3}(s,x_c).G_{c_4}(s,x_c)$$

$$where,\begin{cases} G_{c_1}(s,x_c)=constant \\ G_{c_2}(s,x_c)=\left(\dfrac{1}{g_{m_{01}}(s)}\right)_{s=0} \\ G_{c_3}(s,x_c)=\dfrac{1}{s^{\alpha_c}} \\ G_{c_4}(s,x_c)=\dfrac{1+T_c.s^{\mu_c}}{1+\tau_c.s} \end{cases} \qquad (16)$$

Where: each part of this controller is well detailed in [5]. The design controller is achieved by finding five variables, which are presented by $x_c=\left[G_{c_1},T_c,\tau_c,\alpha_c,\mu_c\right]^T$ where the PSO algorithm is used. Accordingly, the controller tuning parameters are usually carried out by an optimization process which involves a complex search operation. It may lead to a wrong controller if an obtained solution is trapped in local minima. To avoid this problem difficulty, the second contribution of this paper is to propose an analytical way that reduces the search size of the optimization problem shown in (15),(16), where the tuning parameters of the proposed robust fractional controller depend only on three arguments.

Search space reduction of the optimization problem

A similar procedure given in [15] is used to reduce the variable number of our controller. The proposed idea is summarized as follows:

According to figure 3, if the modelling error is well minimized (*i.e.*, $E_m=0$), the open loop transfer function is determined through the inner loop of figure 3. It yields $L_{ref}=G_c.g_{m_{01}}$. The main goal is to ensure, by L_{ref}, the same characteristic behaviours of the open loop reference model given by Eq. (17):

$$G_c.g_{m_{01}}=\frac{\chi_1}{s^{\chi_2}.(\chi_3.s+1)} \qquad (17)$$

Where: χ_1,χ_2, and χ_3 are the new real parameters determined by an adequate optimization algorithm. According to equation (17), the above proposed fractional order reference model has two parts:

i) The first part presents the Bode's ideal transfer function defined by [25]: $L_{ref1} = \frac{\chi_1}{s^{\chi_2}}$,

ii) The second one corresponds to usual first order transfer function given by: $L_{ref2} = \frac{1}{\chi_3 . s + 1}$.

In the controller synthesis step proposed by Wang [11], the open loop transfer function of the reference integer model is given by:

$$L_{Wang} = \frac{w_n^2}{s\left(s + 2\zeta w_n\right)} \tag{18}$$

Where: ζ and w_n denote, respectively, the damping ratio and the natural frequency of the desired open loop. According to equations (17) and (18), if $\chi_1 = \left(\frac{w_n}{2\zeta}\right), \chi_2 = 1$, and $\chi_3 = \left(\frac{1}{2\zeta w_n}\right)$, the Wang's model becomes a particular case of our proposed fractional model. Furthermore, if $\chi_3 = 0$, the ideal Bode's model becomes also a particular case of our proposition. As a result, our proposition has the ability of satisfying a more level of imposed specifications than the one proposed by other synthesis controllers in the standard SP control strategy. Based on equations (16) and (17), the transfer function of the proposed robust fractional controller is rewritten as Eq. (19):

$$G_c = \left[\chi_1\right] . \left[\frac{1}{K_m}\right] . \left[\frac{1}{s^{\chi_2}}\right] . \left[\frac{1 + T_{m_1} s^{\alpha_m}}{\chi_3 . s + 1}\right] \tag{19}$$

Where: the controller parameters are reduced to the three degrees of freedom χ_1, χ_2 , and χ_3. Its optimal values are given by an adequate optimization algorithm, in which some imposed H_∞ specifications are well satisfied.

Fractional-Order SP Controller Based on Optimization Problem

In the robust control theory, the robustness conditions impose certain constraints. These constraints present the magnitudes of sensitivity functions with small values for all frequencies. In addition, the sensor noise and the plant disturbance must have large values
The objective of this issue is to achieve this robustness with minimum magnitudes of the control signal. So, the above conditions are formulated as the following multivariable

optimization problem:

$$\min_{x_c} J_{c_1}(x_c) = \min_{x_c} \max_{\omega} \bar{\sigma}\left[S_d\left(j\omega, x_c\right)\right] \Leftrightarrow \left\|S_d\left(s, x_c\right)\right\|_\infty \le \gamma_1 \quad (20)$$

$$\min_{x_c} J_{c_2}(x_c) = \min_{x_c} \max_{\omega} \bar{\sigma}\left[S_c\left(j\omega, x_c\right)\right] \Leftrightarrow \left\|S_c\left(s, x_c\right)\right\|_\infty \le \gamma_2 \quad (21)$$

$$\min_{x_c} J_{c_3}(x_c) = \min_{x_c} \max_{\omega} \bar{\sigma}\left[K\left(j\omega, x_c\right)S_d\left(j\omega, x_c\right)\right] \Leftrightarrow \left\|K\left(s, x_c\right)S_d\left(s, x_c\right)\right\|_\infty \le \gamma_3 \quad (22)$$

$$\min_{x_c} J_{c_4}(x_c) = \min_{x_c} \max_{\omega} \bar{\sigma}\left[G_r\left(j\omega, x_c\right)S_d\left(j\omega, x_c\right)\right] \Leftrightarrow \left\|G_r\left(s, x_c\right)S_d\left(s, x_c\right)\right\|_\infty \le \gamma_4 \quad (23)$$

Where: S_d and S_c denote, respectively, the sensitivity and the complementary sensitivity functions, with formula $S_d\left(s, x_c\right) = \left[1 + G_r K\left(s, x_c\right)\right]^{-1}, S_c\left(s, x_c\right) = 1 - S_d\left(s, x_c\right)$. In addition, $\bar{\sigma}(*)$ presents the maximum singular value of the transfer function $(*)$. Knowing that, $\bar{\sigma}\left(X_i(s, x_c)\right)$ is numerically equivalent to the infinite norm of $X_i(s,x_c)$, Where $\left\|X_i\left(s, x_c\right)\right\|_\infty \le \gamma_i$, with γ_i is the positive level fixed by the user. In order to avoid the complexity of calculus, and based on the recent property, we tried to simplify the above multi-objective optimization problem to a mono-objective one, under constraints. To achieve this goal, one of the four objective functions (see equations (20)-(23)) must be selected as the main objective function and the others are considered as the inequality constraints. Then, we will solve the obtained optimization problem, using the PSO algorithm [22] to determine the optimal controller parameters vector, Eq. (24):

$$x_c^* = \left(\chi_1^*, \chi_2^*, \chi_3^*\right)^T . \quad (24)$$

Consequently, the equation $\min_{x_c} \max_w \bar{\sigma}\left[X_i\left(j\omega, x_c\right)\right]$ becomes numerically equivalent to $\left\|X_i\left(s, x_c\right)\right\|_\infty \le \gamma_i$, where γ_i is the positive level fixed by the user.

Proposed algorithm for the modelling and the controller synthesis steps

In the modelling step, the fractional model parameters are determined according to the following steps:

Step 1: Choose the set of lower and upper bounds that limit the design variable x_m, in which the following condition should be satisfied: $x_{min} < x_m < x_{max}$. In addition, choose the positive scalar ε_m and go the next step.

Step 2: Notice that the n frequency responses of the actual process $G_r(j\omega_k)_{k=1,\cdots,n}$ are, usually, available in the world control society. In this paper, the previous frequency responses are generated using the Matlab command *logspace*, in which the n logarithmically equally spaced points between decades $10^{\omega_{\min}}$ and $10^{\omega_{\max}}$ can be generated. Set $\ell = 0$ and go the next step.

Step 3: For each ℓ step of the PSO algorithm, the solution vector x_{m_ℓ} is generated within the range $[x_{min}, x_{max}]$. Based upon this solution, the n frequency responses of the proposed fractional model $G_m(j\omega_k)_{k=1,\cdots,n}$ are determined using equation (8). Therefore, the cost function of the optimization problem (11) is determined through the mean absolute error between both gains of the actual process and the proposed model. However, the inequality constraints of the optimization problem (11) are determined by the n phases of the actual process and the proposed fractional model where each obtained value is compared by the positive level ε_m.

Step 4: From the previous optimization problem, the new bounded constraints optimization problem is determined using the same idea proposed in [23].

Step 5: The obtained new optimization problem is solved by the Matlab command *particle swarm*, which is available in the Matlab®2015a\toolbox\globaloptim. The syntax of this command is given by: $x_m = \text{particleswarm}(obj\_fun, n\text{var}, x_{\min}, x_{\max})$, where obj_fun is the M-function that defined the cost function of the bounded constraints optimization problem, $n\text{var} = 5$ is the number of the proposed fractional model.

Step 6: If the stop condition is satisfied, the algorithm terminates with the optimal solution vector $x_{m_{opt}}$. Otherwise, set $\ell \leftarrow \ell + 1$ and go back to the step 3.

The same previous steps should be followed to determine the robust fractional controller parameters, in which the optimization problem can be formulated through equations (20) to (23) where its optimal solution $x_c^* = (\chi_1^*, \chi_2^*, \chi_3^*)^T$ is done by the PSO algorithm.

In the next section, two actual processes with long dead time will be chosen to validate the proposed fractional controller based upon fractional model for the SP control strategy.

Example 1: Actual process with dead time delay

Considering the actual process defined in [11, 15] by: $G_r(s)=\frac{1}{(1+s)^5}e^{-4s}$, the following numerical data are chosen in the modelling phase as:

$$0<K_m<2,\ 0<T_{m_1},T_{m_2}<3,\ 0<\alpha_m<1,\ 0<\theta_m<6,\ n=200,\ \omega\in\left[10^{-4},10^{4}\right]\text{ and }\varepsilon_m=10^{-2}.$$

Where: $K_m, T_{m_1}, T_{m_2}, \alpha_m, \theta_m, n, w$ are, the parameters needed in the PSO optimization method. Notice that the integer model proposed by Wang in [11], for the same previous process, is defined by Eq. (25):

$$G_{m\_Wang}(s)=\frac{1}{0.999+1.64.s}e^{-5.79s} \tag{25}$$

Now, in the synthesis step, the desired robust controller should ensure a good minimization of the sensor noises effect, in which the obtained control signal should be given by a less magnitude. These requirements are formulated as the following constrained optimization problem, Eq. (26):

$$\begin{aligned}&\min_{x_c} J_c(x_c)=\min_{x_c}\max_{\omega}\ \bar{\sigma}\left[kS_d\right]\\ &\text{subject to:}\quad h_c(x_c)=\left\|S_c\left(j\omega_c,x_c\right)\right\|_\infty-\gamma\le 0\end{aligned} \tag{26}$$

Also, the numerical data such as: $0<\chi_1,\chi_2,\chi_3<3$, $\omega_c\in\left[0.1,100\right]$ and $\gamma=0.1$ are used in the initialization step. On the other side, the integer order PID controller proposed by Wang in [11] is given by Eq. (27):

$$G_{c\_Wang}(s)=\frac{5.03s+3.06}{s(s+2.48)} \tag{27}$$

Example 2: Multiple-lags process

Considering the actual process defined in [11] by: $G_r(s)=\frac{1}{(1+s)^{10}}$, the following numerical data are chosen in the modelling phase as:

$$0<K_m<1.5,\ 0<T_{m_1},T_{m_2}<4,\ 1<\alpha_m<2,\ 3<\theta_m<6,\ n=500,\ \omega\in[10^{-4},\ 10^{+4}],\ \varepsilon_m=10^{-2}\ .$$

Notice that the integer model proposed by Das in [8] is given by Eq. (28):

$$G_{m\_Das}(s)=\frac{1}{\left(2.549809s+1\right)\left(2.661456s+1\right)}e^{-5.293009s} \tag{28}$$

Next, the proposed controller should satisfy a good trade-off between the robust stability and the nominal performances. This goal is formulated as the following optimization problem, Eq. (29):

$$\begin{aligned} &\min_{x_c} \; J_c(x_c) = \min_{x_c}\max_{\omega} \; \bar{\sigma}\left[S_d\right] \\ &\text{subject to:} \quad h_c(x_c) = \left\| S_c\left(j\omega_c, x_c\right)\right\|_{\infty} - \gamma \le 0 \end{aligned} \tag{29}$$

To determine the proposed fractional controller, we used the numerical data: $0 < \chi_1, \chi_3 < 1$, $1 < \chi_2 < 2$, $\omega_c \in \left[10^{-4} \; 10^{4}\right]$ and $\gamma = 0.01$ in the initialization step. On the other side, the fractional order PID controller proposed by Das in [8] is given by:

$$G_{c\_Das}(s) = \frac{0.486645s + 0.109355}{s^{0.991345} + 0.754894s^{0.707808}} \tag{30}$$

Results and discussion

Figure 6 compares the obtained performances given by the proposed fractional model with those given by the integer model of Wang.

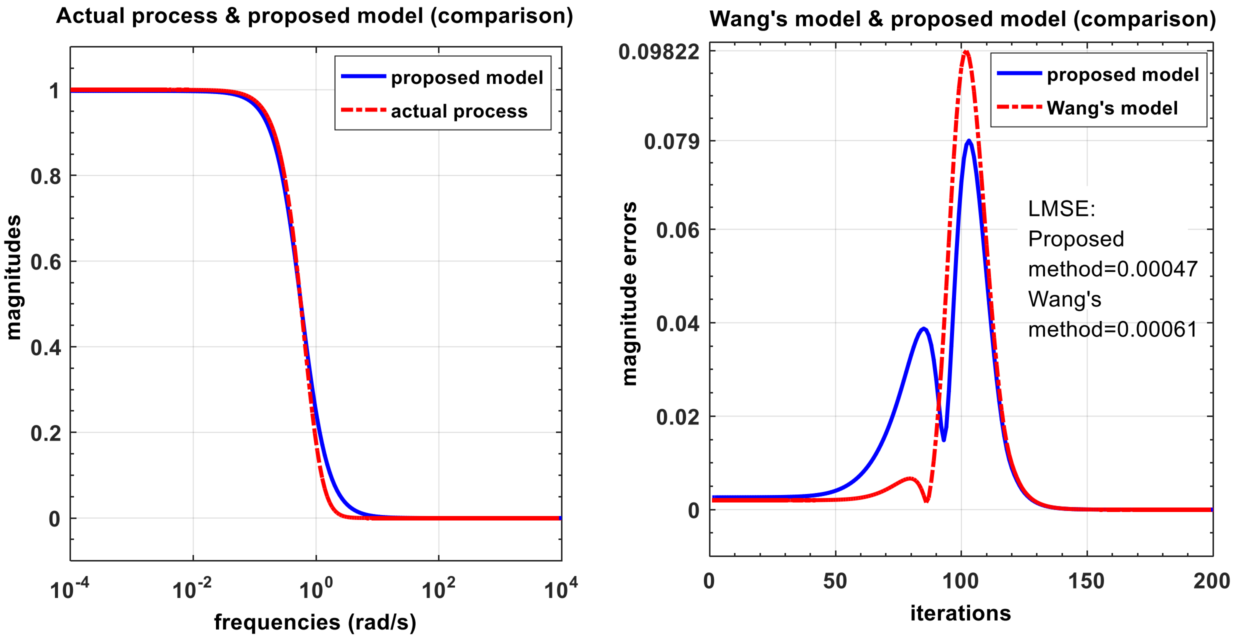

Figure 6. The obtained fractional order model and compared by the Wang's model

The obtained x_m vector needed in the transfer function of the proposed fractional-order model for example 1 is given by equation:

$$x_m = \left[K_m, T_{m_1}, T_{m_2}, \alpha_m, \theta_m\right]^T = \left[0.9974,\; 1.6554,\; 1.8186,\; 0.982,\; 5.7449\right]^T$$

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According to figure 6, it can be seen that the proposed model based on the PSO algorithm ensures a good tracking dynamic to the actual process in each frequency points. Compared to the Wang's model, the proposed modelling method ensures a costly value of the maximum error magnitude $\max|E_p| = 0,079$ and a good minimization of the mean least square error $J_{\min} = 0,00061$. Also, the optimal parameters obtained from running the proposed method are the following $x_c^* = (\chi_1^*, \chi_2^*, \chi_3^*)^T = (0.906,\ 1,\ 0.711)^T$. These parameters are used to obtain the proposed fractional order SP controller for example 1. Figure 7 shows the maximal singular values of both sensitivity functions $\bar{\sigma}[KS_d]$ and $\bar{\sigma}[S_c]$ given by Wang's controller and the proposed one.

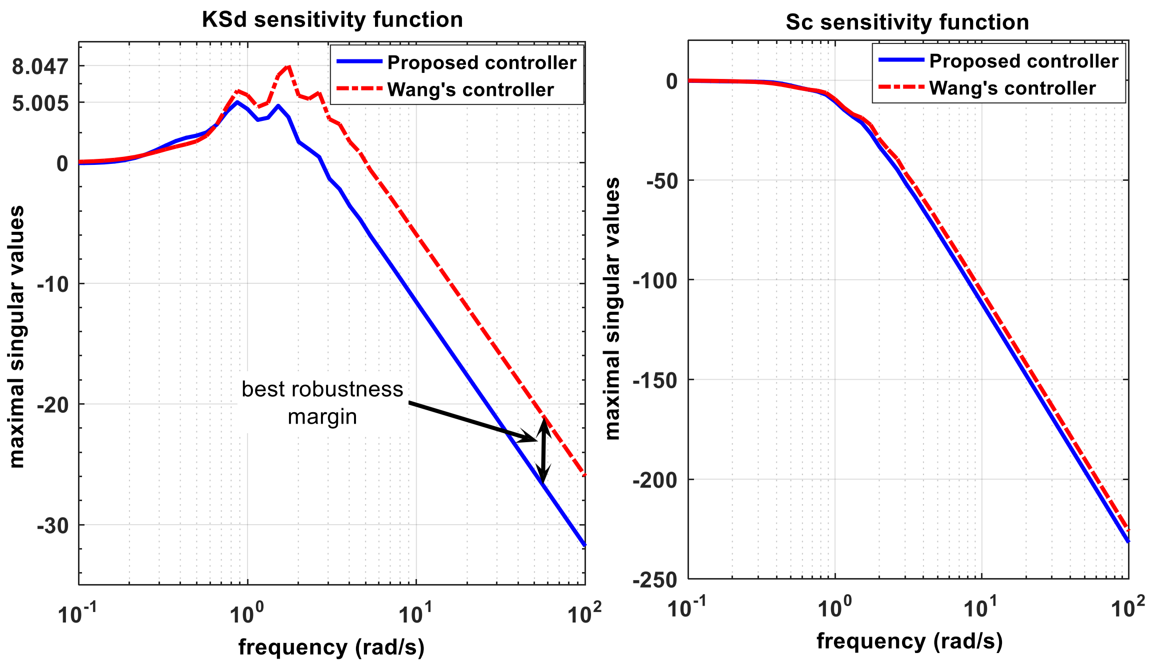

Figure 7. The two sensitivity functions obtained using Wang's method and our proposed one

According to this figure, especially the left side, all the singular values of the sensitivity function KS_d, given by the two synthesis methods, are reduced in all frequency points. This guarantees a proper less control energy. This figure also shows that the better-obtained sensitivity function is reached when its maximum magnitude value is small as much as possible, so that the proposed method ensures a better control signal in the time domain. In addition, it can be seen that, according to the right side of figure 7, the maximum singular values of the complementary sensitivity function S_c are reduced at frequencies that are beyond the system bandwidth in order to secure the robustness at high frequencies. Also, we can see that the better minimization dynamic of the sensor noises effect is given when the maximum singular values of the complementary sensitivity function are as small as possible in high frequencies. As a result, the proposed model controller provides a better robustness margin. To confirm the above results in the time domain, we use the following three exogenous inputs on the feedback control system. The first one represents the set-point reference input Y_{ref} which is assumed as a unit-step function. The second input represents the load disturbance input d_i which is assumed as a unit-step function with a gain equal to -0.25, with start-time 40 seconds. Finally, the third input represents the sensor noise signal η which is taken as a Gaussian random distributed form with mean and variance values are 0 and 0.1 respectively with start-time t = 60 seconds.

Figure 8 compares the obtained performances by the proposed fractional model with those given by the integer Wang's model.

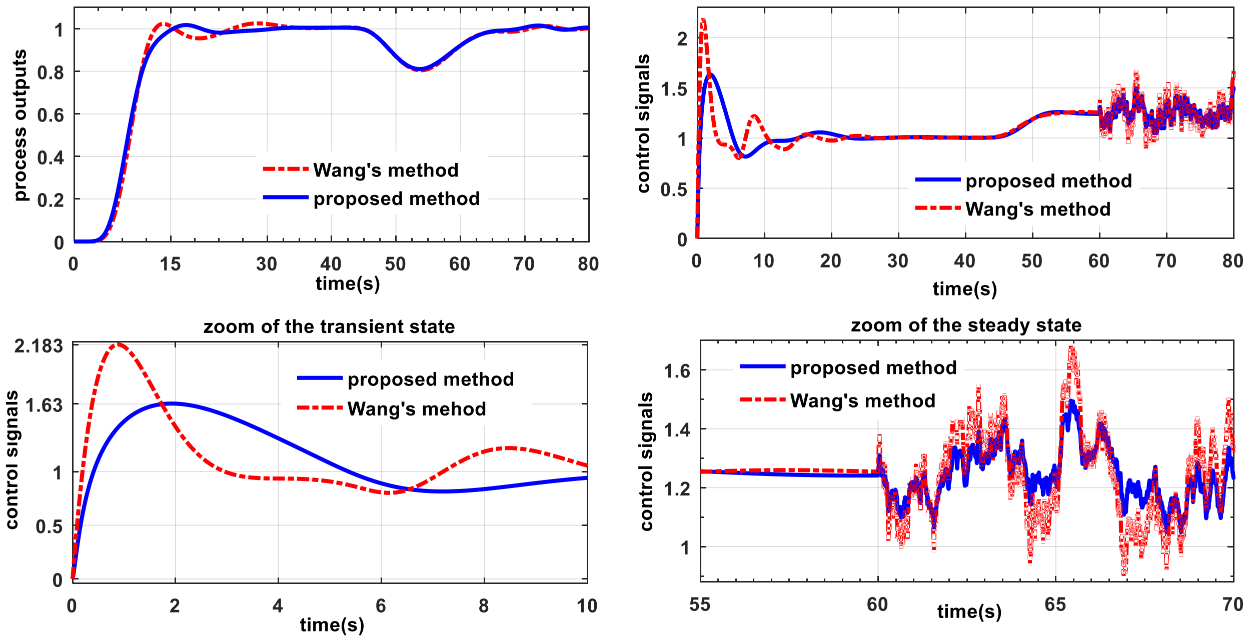

Figure 8. The obtained closed-loop feedback control outputs of the two controllers

From this figure, it is easy to see that the most efficient minimization of the sensor noises effect is ensured by the proposed controller in the steady state. This figure shows also a better control signal with a less magnitude value is guaranteed by the proposed controller in the transient state in which it is not ensured by the other methods.

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Now, after running the proposed algorithm to control the actual process presented in example 2. The optimal x_m vector needed in the proposed fractional-order model is obtained as follows $x_m = \left[K_m, T_{m_1}, T_{m_2}, \alpha_m, \theta_m\right]^T = [1,\ 3.066,\ 3.7112,\ 1.2031,\ 4.6085]^T$. Also, the optimal parameters vector x_c^* of the proposed SP controller (for example 2) is obtained as follows: $x_c^* = \left(\chi_1^*, \chi_2^*, \chi_3^*\right)^T = (0.4304,\ 1.4999,\ 0.1272)^T$. The obtained performances by the proposed controller are compared with those given by Das (see Table 1 and Table 2).

Figures 9, 10, and 11 compare the closed loop performances given by the proposed fractional controller with those given by the fractional order PID controller proposed by Das.

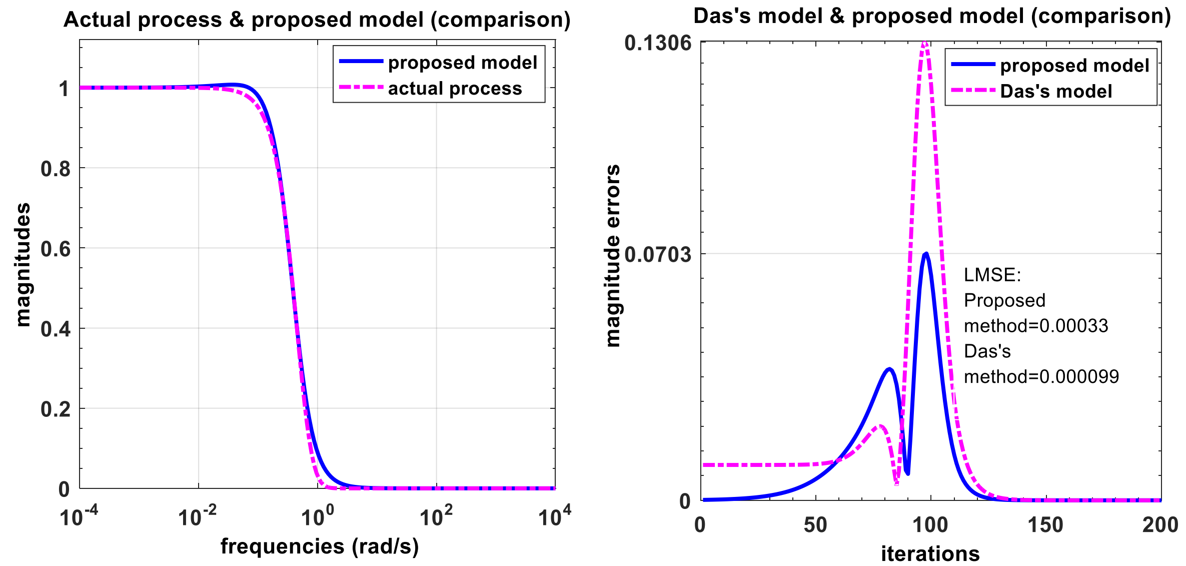

Figure 9.The obtained fractional order model compared by Das's one

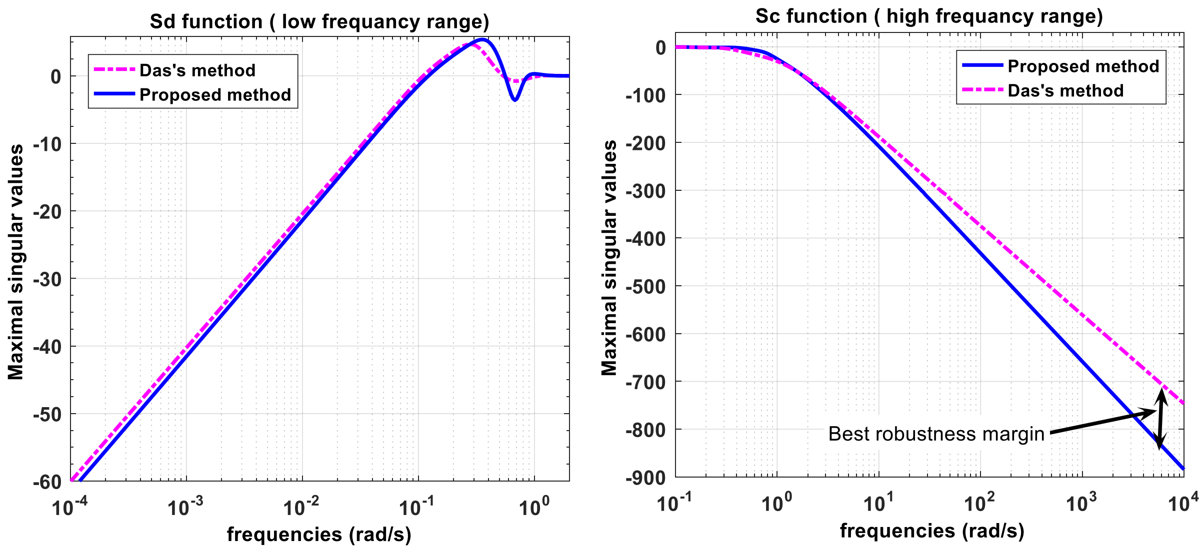

Figure 10. Comparison between the proposed controller and the Das's one in the frequency

domain

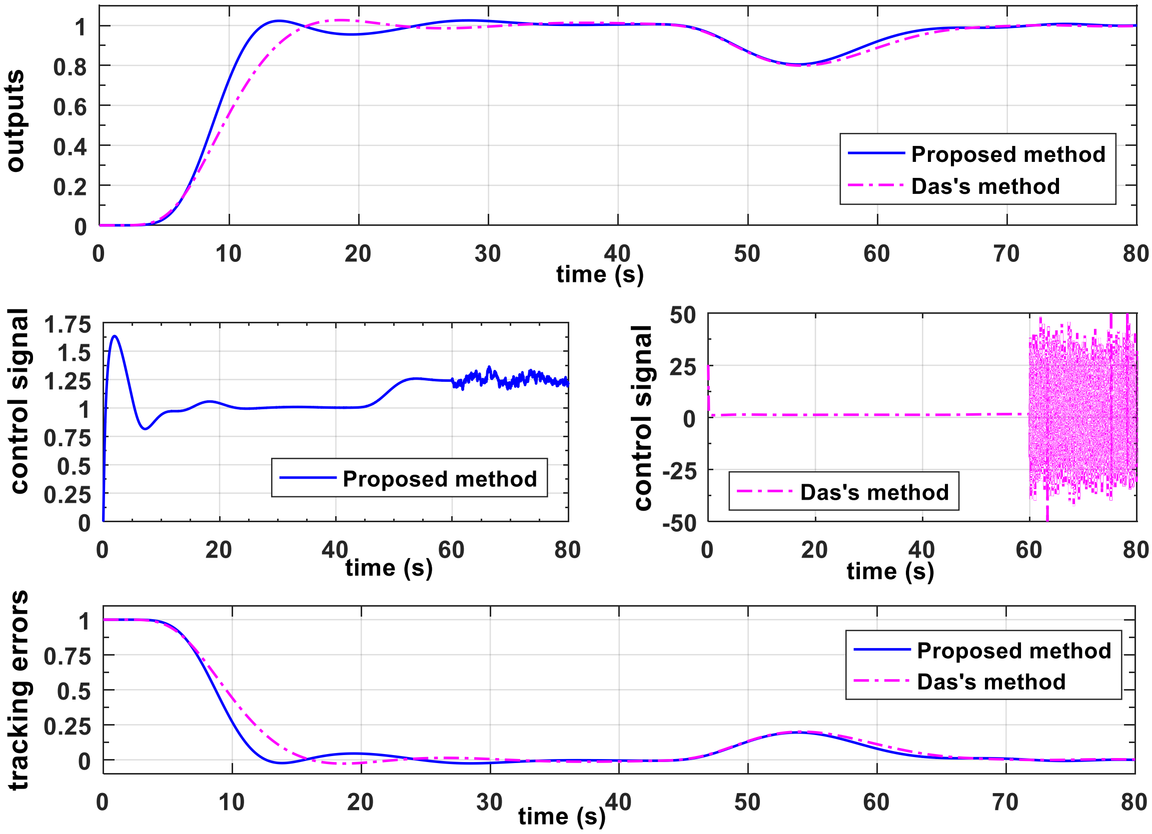

Figure 11.Comparison between the proposed controller and the Das's one in the time domain

Figure 9 compares between both models in the frequency domain for example 2. It can be seen that the proposed model ensures a good tracking dynamic of the multiple lags actual process. This result is explained by a costly value of the maximal error magnitude $\max|E_p| = 0.0703$ and a better minimization of the least mean square error $J_{\min} = 3.3497 \times 10^{-4}$. In the second stage, according to figures 10 and 11, it is easy to see that the proposed controller guarantees the better margin of the stability robustness. This result is explained in the time domain by a good minimization of the sensor noises effect in the steady state. To confirm this result, we apply three exogenous inputs on the feedback control system as follows:

The first one is the set-point reference, which is a unit-step function. The second input corresponds to the load disturbance, which assumed a step function with amplitude -0.25, and the start-time is *t=40* seconds.

Finally, the sensor noise effect signal is chosen as a Gaussian random distributed form, where, the mean and the variance values are respectively, equal to 0 and 0.25, with start-time t=60 seconds. Hence, we can observe that in the steady state, the proposed controller ensures a good minimization of the sensor noise effect. However, in the transient state, the optimal nominal performance is ensured by this proposed controller, where their characteristics are illustrated in table 1 and table 2.

Comparison between the obtained nominal performances by the proposed and the Das's controllers in the transient state (better results are in bold) (Table 1).

Table 1. Comparison proposed and the Das's controllers

Indicators	Proposed controller	Das's controller
Rise time (t_r)	5.2297	7.7493
Settling time (t_s)	63.8253	65.5162
Settling-Min	0.8049	0.7993
Settling-Max	1.0253	1.0268

Table 2. Overshoot and peak time

Indicators	Proposed controller	Das's controller
Overshoot	2.4947	2.9271
Peak at time (s)	1.0253 at (28.4599)	1.0268 at (18.6034)

In control systems design, it is often expected to design a system that has a short rise and settling times, with a small percentage of overshoot or no overshoot at all. Therefore, according to Tables 1 and 2, it is easy to see that the step response of the closed-loop system provided by the proposed method has a good time properties in the transient state, which are a shorter rise time ($t_r = 5.2297$ seconds), faster settling time ($t_r = 63.8253$ seconds) with less overshoot peak.

Conclusions

A new design for a fractional-order controller is proposed combining the Smith Predictor and the fractional reference model. This proposed method is based on the determination of an optimal fractional order model by resolving the proposed bounded constraint optimization problem using PSO algorithm.

In order to satisfy some frequency specifications, a controller synthesis is proposed, where the parameters are obtained by a bounded minimization problem with a reduced search space.

The higher order dead time process has been used to demonstrate the efficiency of the proposed method. The simulation results show that, the proposed controller enhances the performance robustness in both time and frequency domains.

Also, a good tracking behaviour of the set-point references, a good attenuation of the load disturbance dynamic, and a good minimization of both the sensor noises effect, and the energy control cost have been guaranteed.

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